Rational Numbers

If you can write a number as a *ratio of two integers*, it is a **rational number.**

For example, 4.3 is a rational number because we can write it as the ratio $\frac{43}{10}$ or 43:10.

Note: To represent rational numbers, we usually indicate the ratio with a fraction line rather than a colon.

Examples of rational numbers

Since −10 can be written as $\frac{-10}{1}$, it is a rational number. It can also be written as $\frac{10}{-1}$.

Since 0.1 can be written as $\frac{1}{10}$, it is a rational number.

Since 3.24 can be written as $\frac{324}{100}$, it, too, is a rational number.

Negative fractions

The ratio of the integers 7 and −10 gives us the fraction $\frac{7}{-10}$. As we studied earlier, we usually write this as $-\frac{7}{10}$ and read it as "negative seven tenths."

Obviously, all fractions, whether negative or positive, are rational numbers.

Negative fractions give us negative decimals.

For example, $-\frac{8}{10}$ is written as a decimal as −0.8, and $-5\frac{21}{100} = -5.21$.

You can write a rational number as a ratio of two integers in many ways.

For example, the decimal −1.4 can be written as a ratio of two integers in all these ways (and more!):

$$-1.4 = \frac{-14}{10} = \frac{-28}{20} = \frac{28}{-20} = \frac{42}{-30} = \frac{-42}{30} = \frac{-7}{5}$$

So −1.4 is *definitely* a rational number! ☺ But the same holds true for all rational numbers—you can always write them as a ratio of two integers in multitudes of ways.

1. Write these numbers as a ratio (fraction) of two integers.

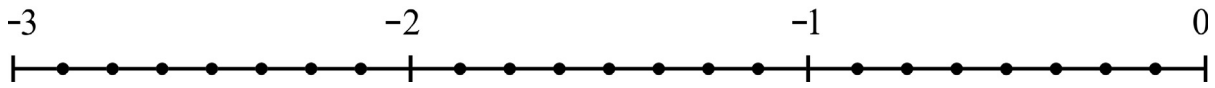
a. 6	**b.** −100	**c.** 0	**d.** 0.21
e. −1.9	**f.** −5.4	**g.** −0.56	**h.** 0.022

2. Are all percents, such as 34% or 5%, rational numbers? Justify your answer.

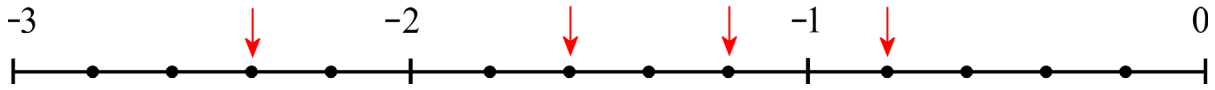
3. Form a fraction from the two given integers. Then convert it into a decimal.

a. 8 and 5	**b.** −4 and 10	**c.** 89 and −100
d. −5 and 2	**e.** 91 and −1000	**f.** −1 and −4

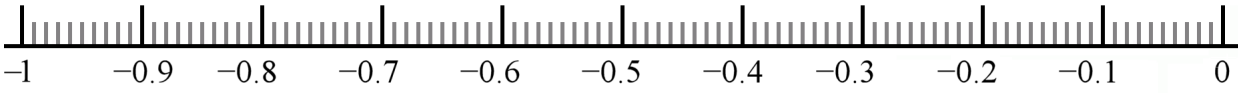
4. Mark the fractions and mixed numbers on the number line below: $-\frac{1}{2}$, $-\frac{7}{8}$, $-1\frac{5}{8}$, $-\frac{9}{4}$, $-2\frac{3}{4}$



5. Write the fractions marked by the arrows.



6. Mark the decimals on the number line: −0.11, −0.58, −0.72, −0.04



7. Sketch a number line from −3 to 0, with tick marks at every tenth. Then mark the following numbers on your number line: −0.2, −1.5, −2.8, −3/5, and −5/2.

8. Write these rational numbers as ratios of two integers (fractions) in a lot of different ways.

a. $-2 = -\frac{2}{1} =$
b. $0.6 = \frac{6}{10} =$

9. Compare, writing < or > in between the numbers.

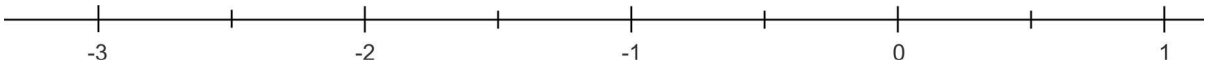
a. $-\frac{7}{8}$ $\quad -1$	**b.** $-\frac{3}{4}$ $\quad \frac{1}{2}$	**c.** $-\frac{15}{2}$ $\quad -7$	**d.** -0.98 $\quad -1.4$

10. Order these rational numbers in order, from the smallest to the greatest.

$2.1 \quad -\frac{1}{8} \quad -1 \quad -\frac{7}{3} \quad -2.01 \quad 1 \quad \frac{1}{3} \quad -0.5$

11. Mark the decimals *and* the fractions on the number line, approximately.

$0.3 \quad -\frac{2}{5} \quad -0.8 \quad -\frac{10}{4} \quad -2.1 \quad -1\frac{1}{2} \quad -\frac{17}{10} \quad 0.95$



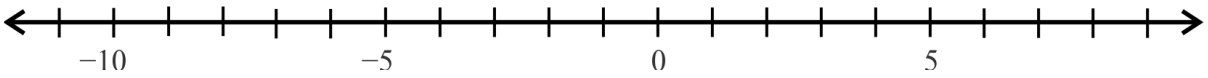
Recall that the absolute value of a number is its distance from zero.

Below, the thickened line shows the set of numbers that are greater than −3 and at the same time, less than 3. We can write it like this: the set of numbers x so that $-3 < x < 3$.

These are the numbers whose absolute value is less than 3, in other words the set of numbers for which $|x| < 3$. Their distance to zero is less than 3. For example, −2.8 and 0.492 and −6/5 belong to this set.

Note that 3 and −3 are not part of this set; that is why we use an open circle at 3 and −3.

12. **a.** Show on the number line the set of numbers x for which $|x| < 1.5$



b. List three rational numbers in that set that are not integers.

13. List three rational numbers r so that $|r| < 2$ and $r > -1$.