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Percentage of a Number (Mental Math)

100% of something means *all* of it. 1% of something means 1/100 of it.

Since one percent means “a hundredth part,” calculating a percentage of a quantity is the same thing as finding a fractional part of it. So **percentages are really fractions!**

How much is 1% of 200 kg? This means how much is 1/100 of 200 kg? It is simply 2 kg.

To find 1% of something (1/100 of something), divide by 100.

Do you remember how to divide by 100 in your head? Just move the decimal point two places to the left. For example, 1% of 540 is 5.4, and 1% of 8.30 is 0.083.

To find 2% of some quantity, first find 1% of it, and double that.

For example, let’s find 2% of \$6. Since 1% of \$6 is \$0.06, then 2% of \$6 is \$0.12.

To find 10% of some quantity, divide by 10.

Why does that work? It is because 10% is 10/100, which equals 1/10. So 10% is 1/10 of the quantity!

For example, 10% of \$780 is \$78. And 10% of \$6.50 is \$0.65.

(To divide by 10 in your head, just move the decimal point one place to the left.)

Can you think of a way to find 20% of a number?

1. Find 10% of these numbers.

a. 700 _____ b. 321 _____ c. 60 _____ d. 7 _____

2. Find 1% of these numbers.

a. 700 _____ b. 321 _____ c. 60 _____ d. 7 _____

3. One percent of Mother’s paycheck is \$22. How much is her total paycheck?

4. Fill in the table. Use mental math.

percentage ↓ number →	1,200	80	29	9	5.7
1% of the number					
2% of the number					
10% of the number					
20% of the number					

5. Fill in this guide for using mental math with percentages:

Mental Math and Percentage of a Number	
50% is $\frac{1}{2}$. To find 50% of a number, divide by _____.	50% of 244 is _____.
10% is $\frac{1}{10}$. To find 10% of a number, divide by _____.	10% of 47 is _____.
1% is $\frac{1}{100}$. To find 1% of a number, divide by _____.	1% of 530 is _____.
To find 20%, 30%, 40%, 60%, 70%, 80%, or 90% of a number, <ul style="list-style-type: none"> • First find _____% of the number, and • then multiply by 2, 3, 4, 6, 7, 8, or 9. 	10% of 120 is _____. 30% of 120 is _____. 60% of 120 is _____.

6. Find the percentages. Use mental math.

a. 10% of 60 kg _____ 20% of 60 kg _____	b. 10% of \$14 _____ 30% of \$14 _____	c. 10% of 5 m _____ 40% of 5 m _____
d. 1% of \$60 _____ 4% of \$60 _____	e. 10% of 110 cm _____ 70% of 110 cm _____	f. 1% of \$1,330 _____ 3% of \$1,330 _____

7. David pays a 20% income tax on his \$2,100 salary.

- How many dollars is the tax?
- How much money does he have left after paying the tax?
- What percentage of his salary does he have left?

8. Nancy pays 30% of her \$3,100 salary in taxes. How much money does she have left after paying the tax?

9. Identify the errors that these children made. Then find the correct answers.

a. Find 90% of \$55. Peter's solution: 10% of \$55 is \$5.50 So, I subtract $100\% - \$5.50 = \94.50	b. Find 6% of \$1,400. Patricia's solution: 1% of \$1,400 is \$1.40. So, 6% is six times that, or \$8.40.
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Some more mental math “tricks”	
90% of a quantity First find 10% of the quantity and then subtract that from 100% of it.	25% of a quantity 25% is the same as $\frac{1}{4}$. So, to find 25% of a quantity, divide it by 4.
12% of a quantity First find 10% of it. Then find 1% of it, and use that 1% to find 2% of it. Then add the 10% and the 2%.	75% of a quantity 75% is $\frac{3}{4}$. First find $\frac{1}{4}$ of the quantity and multiply that by 3.

10. Find percentages of the quantities.

a. 50% of 26 cm _____	b. 25% of 40 mm _____	c. 80% of 45 m _____
d. 75% of \$4.40 _____	e. 90% of 1.2 m _____	f. 25% of 120 kg _____

11. Fill in the mental math method for finding 12% of \$65.

10% of \$65 is \$_____. 1% of \$65 is \$_____. 2% of \$65 is \$_____.

Now, add to get 12% of \$65: \$_____ + \$_____ = \$_____

12. Fill in the mental math shortcut for finding 24% of 44 kg.

25% of 44 kg is _____ kg. 1% of 44 kg is _____ kg.

Subtract _____ kg – _____ kg = _____ kg

13. From her cell phone bill, Hannah sees that of the 340 text messages she sent last month, 15% were sent during the night at a cheaper rate. How many messages did Hannah send at night? During the day?

14. A herd of 40 horses had some bay, some chestnut and some gray horses. Thirty percent of them are bay, and 45% are chestnut. How many horses are gray?

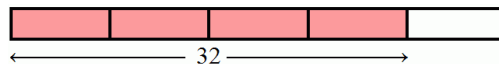
15. A college has 1,500 students, and 12% of them ride the bus. Another 25% walk to the college. How many students do not do either?

Finding the Total When the Percentage Is Known

Use a bar model to find the unknown total when you know the percentage and the quantity.

Example 1. If 32 red marbles make up $\frac{4}{5}$ of the total number of marbles, how many marbles are there in all?

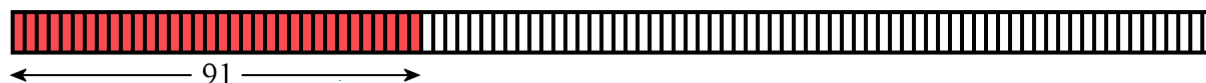
Look at the bar model. We have drawn the marbles as divided into 5 equal “blocks.” Four of those five blocks make up a total of 32 marbles. So, one block, or $\frac{1}{5}$ of the marbles, is 8 marbles. From that it is easy to calculate the total: $5 \cdot 8 = 40$ marbles.



The same reasoning works if the part of the marbles is given as a *percentage* instead of as a fraction:

Example 2. If 91 red marbles is 35% of the total number of marbles, how many marbles are there in all?

In the model, we need 100 little “blocks” with 35 of them colored (since $\frac{35}{100}$ of the marbles are red.)



The calculation is done the same way: If 35 “blocks” or 35% make up 91 marbles, then one “block”, or one percent, is $91 \div 35 = 2.6$. Then, to find the total, simply multiply that number by 100: $2.6 \cdot 100 = 260$.

1. Margie gave away 40 marbles, which was 20% of the marbles that she had.

How many marbles did Margie have at first?

Hint: Instead of 100 blocks, you can use 5 blocks, each representing 20% or $\frac{1}{5}$.

2. Emma cut down the amount of sugar in a recipe by 75%.

Now, she uses only $\frac{1}{2}$ cup of sugar.

How much sugar did the recipe call for originally?

Hint: Instead of 100 blocks, you can use 4 blocks, each representing 25%.

3. When Eric bought a guitar for \$90, he used up 12% of the money he had.

How much money did he have at first?

Example 3. A phone was discounted by 40% and now costs \$72. What was the price before the discount?

The cost now, \$72, represents **60%** of the original total—not 40%.

We can find 10% of the original price by dividing $\$72 \div 6 = \12 . And from that, 100% of the price is 10 times that, or \$120. If this confuses you, draw a bar model with 10 parts, each representing 10% of the original price.

4. A dress was discounted by 20%.

The discounted price is \$24.

What was the price before the discount?

5. A concert ticket was discounted by 60%.

The discounted price is \$21.60.

What was the original price?

6. Joe spent 72% of his money, and now he has \$56 left.

How much did Joe have to begin with?

7. Crystal spent 52% of her money and now she has \$120 left.

How much did she spend?



8. Uncle Jack raises two different breeds of cows on his ranch. Of his cows, 28% are Black Angus and the rest are Hereford. If he has 420 Black Angus cows, how many Herefords does he have?



9. A survey found out that 16% of the people who had bought a certain brand of coffee grinder were unhappy with it. If there were 126 people who *were* happy with it, then how many people in total had bought that brand?



Puzzle Corner

One calculator is discounted by 30% and now costs \$42.
Another is discounted by 25% and now it also costs \$42.
Which calculator had the cheaper original price? How much cheaper?

Percent Equations and Price Changes, Part 1

Percents are fractions with a denominator of 100, which means we can use decimals to represent them. For example, $60\% = 60/100 = 0.6$. Using decimals is especially handy when writing equations from percent problems.

Example 1. Let p be the price of an item. How can we write 20% of p as a mathematical expression? Recall that “of” relates to multiplication:

$$\begin{array}{ccc} 20\% & \text{of} & p \\ \downarrow & \downarrow & \downarrow \\ 0.2 & \cdot & p \end{array}$$

So, 20% of p is the same as $0.2p$.

Example 2. Let p be the price of an item. What common real-life situation does the expression $p + 0.14p$ represent?

The expression $p + 0.14p$ is really $1p + 0.14p$ and simplifies to $1.14p$.

It shows that 14% of the price p is added to the price itself. This signifies the price of an item was increased by 14%.

You may use a calculator for all the problems in this lesson, except question #9.

1. Write the mathematical expression for each phrase.

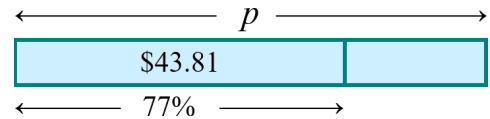
Phrase	Expression
price	p
34% of the price	
1.5% of the price	
300% of the price	

Phrase	Expression
cost	C
5.6% of the cost	
105% of the cost	
105.5% of the cost	

2. **a.** An item with price p is discounted by 20%. Write an expression to represent the new, discounted price.
- b.** Let's say the new, discounted price is \$279.20. Use the expression you wrote in (a) and now write an equation for the purpose of finding the original price, p . Then solve your equation.
3. The price of an item is increased by 12%. Which expressions below represent the new price of the item?
- ☐ $0.12p$
☐ $1.88p$
☐ $1.12p$
☐ $p + 0.12p$
☐ $0.88p$
4. The price of an item is increased by 12%, and now it costs \$85.80. Write an equation to find the price before the increase and solve it.

5. The rent was increased by 5% and is now \$215.25. What was the rent before the increase?
Write an equation for this situation and solve it.

6. A handbag is on sale for 23% off and now costs \$43.81.
How can you use the bar model to find the original price using *logical reasoning* (not an equation)?



7. Serena is solving the math problem on the right. She says, “I think the correct equation is $2(p - 0.3) = 98$. Because, we first subtract the 30% of the price p , (and 30% is 0.3) and then multiply that price by 2, and get \$98.”

a. Explain why that equation is the wrong choice.

b. Solve the problem.

A tablet is discounted by 30%. Matthew bought two of them, and paid \$98.

Which equation(s) below can be used to find the price of the tablet before the discount (p)?

$$2(p - 30) = 98$$

$$2p - 30 = 98$$

$$0.7p = 49$$

$$2(p - 0.3p) = 98$$

$$2(p - 0.3) = 98$$

8. Alice bought five bottles of hair conditioner at 15% off.
Her total bill was \$50.79. What was the price of one bottle of hair conditioner *before* the discount?

Sales tax / VAT

Most countries and states charge a value added tax (VAT) or a sales tax on every purchase made. The tax rate varies a lot among different places and items, and can be as low as 0% and as high as 27% (Hungary).

For example, if the sales tax is 6.5%, and the base price of an item is \$20, the customer has to pay the \$20 *plus* 6.5% of \$20. The final price is therefore $\$20 + 0.065 \cdot \$20 = \$20 + \$1.30 = \$21.30$.

The fastest way to calculate the final price is actually with the single multiplication $1.065(\$20) = \21.30 . (The \$20 in parentheses indicates multiplication. In other words, $1.065(\$20)$ is the same as $1.065 \cdot \$20$.)

9. Find the final price when the base price and the sales tax rate are given. This is a mental math workout, so do not use a calculator!

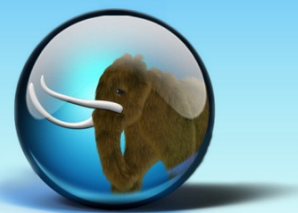
a. Bicycle: \$100; sales tax 7%. Tax to add: \$ _____ Price after tax: \$ _____	b. Fridge: \$400; sales tax 6%. Tax to add: \$ _____ Price after tax: \$ _____	c. Haircut: \$50; sales tax 3%. Tax to add: \$ _____ Price after tax: \$ _____
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10. Find the final price of a music CD with a base price of \$21.50 when the sales tax is 6.7%.
11. A printer is discounted by 10%. The sales tax is 6%. Ashley pays \$290.02 for the printer. What was its price before the discount and the tax?
12. A misting fan is discounted by 15% and then the *already* discounted price is further discounted by 20%.
- a.** If the original price is p , write an expression to represent the final price, after the two discounts.
- b.** After the two discounts, is the fan discounted by 35% from the original price?

Puzzle Corner

A dress is discounted by 10% and then the already discounted price is further discounted by 15%. Now the dress costs \$35.57. What was its price originally? Write an equation and solve it.

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